

OPTIMAL POWER FLOW AND PRICES IN THE ELECTRICITY MARKET USING THE HYBRID PPSOGSA ALGORITHM

Milena Jevtić¹, Miroljub Jevtić², Jordan Radosavljević², Sanela Arsić¹,
Dardan Klimenta²

¹Technical Faculty in Bor, University of Belgrade, Bor, Serbia

²Faculty of Technical Sciences Kosovska Mitrovica, University of Pristina in Kosovska Mitrovica,
Serbia

mjevtic@tf.bor.ac.rs

Abstract

In this paper, the PPSOGSA algorithm is proposed to optimize the nodal prices and power flows transacted between the tiers of the supply chain (SC) in a deregulated electricity market. The hybrid PPSOGSA algorithm is a combination of phasor particle swarm optimization (PPSO) and gravitational search algorithm (GSA). The equilibrium model of SC was applied. The objective function is the function of total profit of participants in the SC. The applied cost functions of participants are nonlinear and non-separable. The results of PPSOGSA application are compared with the results of the modified projection method (MPM) for the numerical solution of the variational inequality of SC, which is applied in the literature for solving the same problem, and with the results of genetic algorithm (GA) as one of the basic meta-heuristic algorithms. The results showed that PPSOGSA gives the best results compared to MPM and GA. Moreover, in the case of PPSOGSA application the equilibrium conditions are fully satisfied while in the case of MPM the equilibrium conditions are satisfied with a small error. It is found that PPSOGSA converges in the much lesser number of iterations and gives better results than the MPM. This new application of PPSOGSA enables the handling of decision makers, who operate in the electricity market and optimize energy flow and prices, minimize payment cost and maximize profit.

Keywords: meta-heuristics, particle swarm optimization, electricity market, supply chain

INTRODUCTION

The equilibrium model of supply chain (SC) [1 - 3] is often used to analyze competitive oligopolistic markets [4, 5]. Equilibrium in this case refers to a condition where a market price is determined through competition such that the amount of goods sought by buyers is equal to the amount of goods produced by sellers. In [6] the equilibrium model of SC was applied to the analysis of the deregulated electricity market. First, based on the equilibrium state in the supply chain, the variational inequality [7] was derived. A variational inequality is an inequality which has to be solved for all possible values of a given variable, belonging usually to a convex set. Then, using the algorithm for numerical solution of the variational inequality, the equilibrium flows of transacted powers and equilibrium nodal

prices in SC were calculated. This algorithm, called the modified projection method (MPM) [8, 9], is suitable for solving the network equilibrium problems because of his good form for computational application. In [1, 6, 7] the MPM was applied to several numerical examples for solving flows and prices in the competitive SC network using the equilibrium model. In these examples the MPM converged in 230 to 633 iterations.

In recent years, the meta-heuristic algorithms were used to solve many complex large-scale nonlinear problems instead of mathematical methods in order to shorten the computation time and obtain a solution with sufficient accuracy. In this paper, the recently developed hybrid meta-heuristic algorithm PPSOGSA [10] which combines phasor particle swarm optimization (PPSO) [11] and gravitation search algorithm (GSA) [12] is

applied to solve the same problem as in [6] and [7]. Then, the obtained results are compared with the results of the application of MPM in [6] and [7] and with the results of the application of genetic algorithm (GA) [13] in this paper. The best results were obtained by using the PPSOGSA. The GA has been successfully applied to solve the optimization problems of SC in [14 – 16]. The GSA has The PPSOGSA is one of newest meta-heuristic algorithms and has been successfully applied for solving the optimal power flow problem in power system [10]. In [10] it was shown that the PPSOGSA has a good global search capability, the high speed of convergence and great robustness.

PROBLEM FORMULATION

The general scheme of electricity SC in a deregulated market can be drawn as in Figure 1 [6]. From Figure 1 can be seen that the generators are associated with the power suppliers. In this model the power suppliers represent the power marketers, retailers and brokers. The suppliers buy electric power (energy) from the generators over the spot market on the wholesale market or using the contracts. The consumers at the demand market buy electric power from the suppliers. To deliver the purchased electric power, suppliers pay service to provider, for transmission and distribution of electricity. The scheme of SC (Figure 1) includes three tiers with many nodes. In the first tier of SC this is a total of G generators, in the second tier there are S suppliers and in third tier there are D demand markets. The links between nodes are not physical. The links represent the rights of participants in the SC to buy or sell electricity. Links: $1, \dots, q, \dots, Q$ are variants of the transmission systems through which the power is transferred.

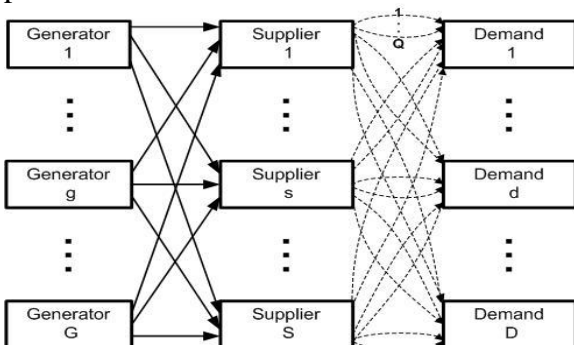


Fig. 1. Scheme of the SC

The function of generator profit ($PROF_g$) may be expressed as follows:

$$PROF_g = \sum_{s=1}^S p_{gs} P_{gs} - F_g(P_g) - \sum_{s=1}^S C_{gs}(P_{gs}) P_{gs} \geq 0, \forall s \quad (1)$$

where indexes g and s denote the generator and supplier, respectively; P_g is the power that is produced from generator g ; P_{gs} is the power that is transferred between generator g and supplier s ; p_{gs} is the price for transfer of the power from generator g to supplier s in the SC; C_{gs} is the cost of activity at generator g for transfer the power from generator g to supplier s (transaction cost); F_g is the production cost of generator g .

In a similar way, the function of supplier profit ($PROF_s$) is:

$$PROF_s = \sum_{d=1}^D \sum_{q=1}^Q p_{sdq} P_{sdq} - C_s(P_{gs}, P_{sdq}) - \sum_{g=1}^G p_{gs} P_{gs} - \sum_{g=1}^G C'_{gs}(P_{gs}) - \sum_{d=1}^D \sum_{q=1}^Q C_{sdq}(P_{sdq}) \quad (2)$$

where index d denotes the consumer; P_{sdq} is the power that is transferred between the supplier s and consumer d via the transmission system q ; C_s is the operating cost of the supplier s ; C_{sdq} is the cost of activity at supplier s for transfer of the power from the supplier s to the consumer d via the transmission system q ; C'_{gs} is the cost of activity at the supplier s for power transfer from the generator g to the supplier s ; p_{sdq} is the price (at supplier s) for the power transmission from the supplier s to the consumer d via the transmission system q ; p_d is the price of the power at consumer d .

For each supplier s there are the constraints:

$$\sum_{d=1}^D \sum_{q=1}^Q P_{sdq} = \sum_{g=1}^G P_{gs} P_{gs} \geq 0, \forall g, P_{sdq} \geq 0, \forall s \quad (3)$$

For each demand market d there is a balance of the power:

$$P'_d(p_d) = \sum_{s=1}^S \sum_{q=1}^Q P_{sdq}, p_d > 0 \quad (4)$$

where P'_d is the power at the consumer d , that depends on the price p_d at the consumer d . The function $P'_d(p_d)$ is usually obtained by approximating the experimentally demand curve [4]. The second tier price p_{sdq} associated with the power supplier s can be expressed as:

$$P_{sdq} = P_d - c_{sdq} \quad (5)$$

where c_{sdq} is the unit transaction cost between the power supplier s and the consumer d via the transmission system q .

The price p_{gs} is determined from the Nash equilibrium [17] that underlying the non-cooperative behavior of generators. It means that each generator will determine his optimal power generation given the optimal ones of the competitors. Based on this concept, the following expression can be derived from (1):

$$P_{gs} = \frac{\partial F_g(P_g)}{\partial P_{gs}} + \frac{\partial C_{gs}(P_{gs})}{\partial P_{gs}} \quad (6)$$

The interpretation of (6) is as follows: the price p_{gs} is equal to the sum of the marginal production cost (first term on the right) plus marginal transaction cost of the generator, associated with that supplier (second term).

The units of variables in this model are: MW (for power flows), \$/h (for profits, prices and costs).

SOLUTION METHOD

The PPSOGSA is a hybrid algorithm which combines the algorithms: PPSO and GSA. In relation to the basic PSO [18] and PPSO, the PPSOGSA combines the ability for social thinking (gbest) in PSO and PPSO with the local search capability of GSA. In the PPSOGSA, the control parameters (c_1 and c_2) of the PSO are modeled with the phase angle (θ), inspired by phasor theory. The general structure of the PPSOGSA algorithm is as follows:

Initialization

1. Define the objective function $F(\mathbf{x}_i)$ and a space of possible solutions \mathbf{X} .
2. Generate the number of search agents (N) (possible solutions) which are randomly selected between the limits of the control variables values. Set the iteration counter $t=1$.

Calculation and iterative procedure

3. Calculation the fitness value for each agent in the current population.
4. Generate the new population of agents, updating the current velocity and current position of agents, using the following algorithmic operators:

$$\mathbf{v}_i(t+1) = r_1 \cdot \mathbf{v}_i(t) + r_2 \cdot |\cos \theta_i(t)|^{2 \cdot \sin \theta_i(t)} \cdot \mathbf{a}_i(t) + r_3 \cdot |\sin \theta_i(t)|^{2 \cdot \cos \theta_i(t)} \cdot (\mathbf{gbest}(t) - \mathbf{x}_i(t)) \quad (7)$$

$$\mathbf{x}_i(t+1) = \mathbf{x}_i(t) + \mathbf{v}_i(t+1) \quad (8)$$

The phase angle θ_i is selected through uniform distribution $U(0, 2\pi)$ and corresponds to the randomly selected x_i . The phase angle is updated using the following equation:

$$\theta_i(t+1) = \theta_i(t) + |\cos \theta_i(t) + \sin \theta_i(t)| \cdot 2\pi \quad (9)$$

In (7) **gbest** represents the best position (solution) among all best positions of agents, which have been achieved so far; r_1 , r_2 , and r_3 are random numbers in $[0, 1]$; and \mathbf{a}_i is the acceleration of agents and it is updated using the equations in [12].

5. The process of updating the velocities and the positions is stopped by meeting a stop criterion.
6. Report the best solution. Stop.

APPLICATION OF PPSOGSA FOR SOLVING POWER FLOW AND PRICING

Application of PPSOGSA for solving power flow and pricing in the electricity market using above equilibrium model of SC can be described in the following steps:

Step 1. Read the input data: SC configuration, cost functions of generators, suppliers and transaction, and demand functions.

Step 2. Specify the control and depended variables. Specify the limits of variables. Specify the objective function.

Step 3. Set the number of search agents (population size). Set the maximum number of iterations. Generate the initial random search agents.

Step 4. Calculate the values of the objective function (fitness values) for each agent of the current population.

Step 5. Apply the PPSOGSA operators (7), (8), (9) to create a new population of agents which are improved solutions of problem.

Step 6. Repeat steps 4-5 until the stop criterion is meeting, i.e. the maximum number of iterations.

Step 7. Report the optimal solutions.

The PPSOGSA is implemented in MATLAB R2017a computing environment and run on a 1.6 GHz, PC with 3.0 CD RAM. The population size and maximum number of iterations are set to 50 and 200, respectively.

RESULTS OF SIMULATION

The PPSOGSA has been tested on the SC numerical example which was used in [6, 7]. The power generating, transaction, and operating cost functions, and the demand functions of the numerical example are given in Table 1. The SC numerical example includes 3 generators, 2 suppliers and 3 demand markets. The objective is to maximize the total profit of SC ($PROF_g + PROF_s$). The obtained optimal values of power transacted in the SC and demand prices are listed in Table II.

Figure 2 shows the convergence profile of PPSOGSA and GA in solving this problem. It may be observed from this figure that both algorithms, PPSOGSA and GA, converge to their global optimum in 15 iterations. The MPM which was applied in [6, 7] was converged in 232 iterations for same test example.

The results shown for the PPSOGSA and the GA are the best values obtained over 30 consecutive runs. Table 3 shows the comparison of minimum, maximum and mean of the results obtained by PPSOGSA and by GA over 30 runs. It can be seen that the proposed PPSOGSA provides better solutions than GA.

From Table 2 follows that the MPM proposed in [6] and [7], resulted in lesser values of maximum profit than the PPSOGSA and GA which are applied in this work. However, the error of equilibrium condition (3) is zero in the case of PPSOGSA and GA application while that error is $8 \cdot 10^{-4}$ MW in the case of MPM. This means that the application of the PPSOGSA provides more accurate results and the higher maximum profit for the lesser number of iterations than the application of the MPM.

Table 1. Numerical example [6, 7], for simulation

Power generating cost functions, $F_g(P_g)$	$F_1(P_1, P_2) = 2.5P_1^2 + P_1P_2 + 2P_1$ $F_2(P_1, P_2) = 2.5P_2^2 + P_1P_2 + 2P_2$ $F_3(P_1, P_3) = 0.5P_3^2 + 0.5P_1P_3 + 2P_3$
Transaction cost functions faced by the power generators and associated with transacting with the power suppliers, $C_{gs}(P_{gs})$	$C_{11}(P_{11}) = 0.5P_{11}^2 + 3.5P_{11}$ $C_{12}(P_{12}) = 0.5P_{12}^2 + 3.5P_{12}$ $C_{21}(P_{21}) = 0.5P_{21}^2 + 3.5P_{21}$ $C_{22}(P_{22}) = 0.5P_{22}^2 + 3.5P_{22}$ $C_{31}(P_{31}) = 0.5P_{31}^2 + 2P_{31}$ $C_{32}(P_{32}) = 0.5P_{32}^2 + 2P_{32}$
Operating cost functions of the suppliers, $C_s(P_{gs})$	$C_1(P_{g1}) = 0.5(\sum_{g=1}^2 P_{g1})^2$ $C_2(P_{g2}) = 0.5(\sum_{g=1}^2 P_{g2})^2$
Unit transaction costs between the power suppliers and the consumers, c_{sdq}	$c_{111} = q_{111} + 5, c_{121} = q_{121} + 5,$ $c_{131} = q_{131} + 5, c_{211} = q_{211} + 5,$ $c_{221} = q_{221} + 5, c_{231} = q_{231} + 5$
Power of demand, P_d , as a function of the price p_d at demand market	$P'_1(p_1, p_2) = -2p_1 - 1.5p_2 + 1100$ $P'_2(p_2, p_2) = -2p_2 - 1.5p_1 + 1100$ $P'_3(p_1, p_3) = -2p_3 - 1.5p_1 + 1200$

CONCLUSION

The SC can have a complex structure with real processes containing very large numbers of continuous and discrete interrelated variables. Application of the conventional mathematical optimization methods in such SCs may be impossible or may require a long computation time. Therefore, in these cases the meta-heuristic algorithms in which the search of optimal solutions is done on the basis of natural laws, can be successfully applied. In this paper, the PPSOGSA, one of the latest population based algorithms, was successfully applied to solve the optimal power flow and pricing in the SC of the deregulated electric power market. It was maximized the total profit of SC. It was used the equilibrium model of SC in which the participants of the market operate in a competitive manner. The results of PPSOGSA application were compared with the results given in the published literature for same model applying the MPM, one of numeric methods, and with the results obtained by the GA, one of the well-known meta-heuristic algorithms. The PPSOGSA gave the solutions of: maximum profit, the nodal prices and the power flow transacted between the tiers of the SC that are better than the solutions obtained by MPM and GA, in terms of convergence speed, global optimality, solution accuracy, and algorithm reliability. The results of this paper can be used in the formation of the integrated SC in a deregulated electric power market and in the market management.

Table 2. Comparison of the simulation results

Control variable	PPSOGS A	GA	MPM [6]
P_{11} (MW)	12.51392	14.20907	14.2762
P_{12} (MW)	12.51392	14.20907	14.2762
P_{21} (MW)	12.51392	14.20907	14.2762
P_{22} (MW)	12.51392	14.20907	14.2762
P_{31} (MW)	59.97922	56.58187	57.6051
P_{32} (MW)	59.97922	56.58187	57.6051
P_{111} (MW)	20.00244	20.00000	20.3861
P_{121} (MW)	20.00244	20.00000	20.3861
P_{211} (MW)	20.00244	20.00000	20.3861
P_{221} (MW)	20.00244	20.00000	20.3861
P_{131} (MW)	45.00223	45.00000	45.3861
P_{231} (MW)	45.00223	45.00000	45.3861
p_1 (\$/h)	302.85575	302.85714	302.6367
p_2 (\$/h)	302.85575	302.85714	302.6367
p_3 (\$/h)	327.85596	327.85714	327.6367
Max.profit, (\$/h)	15765.2044	15476.0587	15218.6668
Error of equilibrium condition (3), MW	0	0	$8 \cdot 10^{-4}$

Table 3. Statistical evaluation of the simulation results

	min	max	mean	StD
PPSOGSA	15731.1855	15765.2044	15748.1949	$2.8192 \cdot 10^{-8}$
GA	15433.5862	15476.0587	15454.8224	$1.6378 \cdot 10^{-7}$

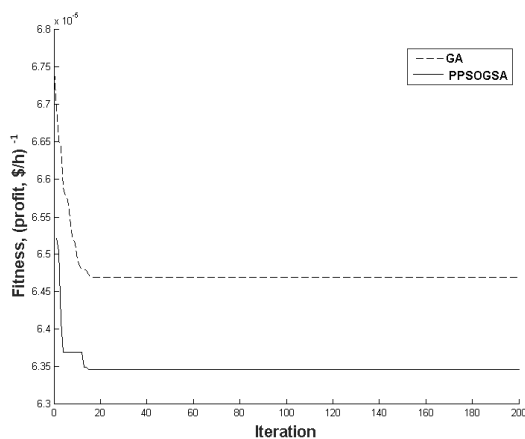


Fig. 2. Convergence of PPSOGSA and GA

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